New framework for determining physical properties - II: properties of photons

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The mathematical forms which describe the structures of all the basic electromagnetic particles, leptons and photons alike, are discussed in this article. Specifically, an equation is presented which explains the value of the Planck Constant $h$, 6.626,075,5x$10^{-34}$kgm$^2$/s. This equation has a general form very similar to those which were discovered that predict the masses of the leptons. From a mathematical view, only a few modifications are required to go from the leptons to the photons and vice versa. The form of this photon equation contrasted with the general form of the lepton equations points toward explanations of the notable physical property differences between the photons and the leptons, e.g. the photons have no mass, display no charge, and have a spin different from the leptons, et cetera. The nature of this photon equation and the geometry that it represents has several profound implications for cosmology and particle physics.

Keywords: leptons, photons, elementary charge, particle masses, Planck Constant, particle structures, computational physics, applied mathematics

I. INTRODUCTION

A. Objective & Scope

The general objective of this paper is to show observed mathematical organizing principles and patterns which describe the structures of the elementary electromagnetic particles. The specific objective of this work is to show a derivation for the Planck constant $h$. This derivation is based on the known geometric structural form of photons, the cylindrical helix. An objective is to show why the Planck Constant $h$ is constant while the photons’ wave lengths can vary over a known range of many orders of magnitude. Additional objectives are to compare the equations found for the photons with those which were found for the mass densities of the leptons, in [1] "On Electromagneric Particles". This mathematical analysis will point toward explanations of the notable physical property differences between these two particle species.

This work is best described as a mathematical framework and demonstration. This framework does not specifically support any particular theory and is not derived from theory. While there may be implications concerning particle theories indirectly supported by this work, only those conclusions directly supported by the equations found will be reported. The equations found are simply presented, not derived, nor proven in the formal sense of those words.

B. Historical and Current Particle Research Efforts

Examining major physics journals such as Physical Review D, Nuclear Physics B, Physics Letters B, Progress in Particle and Nuclear Physics over the last 10, 20, and even 30 years one will find hundreds and even thousands of article focusing on the fermions. Specifically the leptons and quarks are analyzed from about every possible theoretical, mathematical, and experimental angle possible. In the near past much effort centered around the weak force species of bosons. Currently much speculation is devoted to the neutrinos [2-18].

What are conspicuous by their absence are reports of research work devoted to examining and explaining the photons. With the conceptualization and quantification of the Planck Constant, all interest in the photons appears to have dropped. Additionally, it has long been shown that the photons have some form of structure which rotates as it progresses forward, making the outline of a cylindrical helix as it passes an observer. This structure has an electrical and a magnetic vector at right angles to each other, which are also oriented in a radial manner sideways to the photon’s flight path. With this basic knowledge amassed, now the photons are apparently only studied for their use in relation to practical applications, such as polarized sunglasses or lasers. Photons have also been involved in searches for other basic physics knowledge such as with beam splitters examining the question of super-luminal information transfer. But the study of the photon itself for its own sake appears to be a thing of the past.

This is an unfortunate state of affairs because there are still several major unexplained aspects or assumptions made about the photon, one of the most basic of all physics particles. First and foremost, the Planck Constant has never been explained but is treated as a basic assumption. Much effort is made to explain the unexplained masses of the other elementary electromagnetic particles, the leptons, but none to explain this energy equivalent for the photons. Secondly, there is the obvious question, how can the Planck Constant be constant when the wave length of the photons’ varies over a known range of many orders of magnitude?

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II. OUTLINE OF WORK

What we examine in this article is common mathematical structural phenomena which can explain the contained energy of both of the elementary electromagnetic particle species. We use the word energy here to mean a measure of the entrapped or enclosed gravitational energy stabilized by the particle. We develop a tit-for-tat mathematical picture for the gravitational structure of the photons comparable with that discussed in [1] which lead to the precise calculation of masses of the leptons.

What we find amazing here is not only that structural equations can be developed which lead to the exact calculation of the masses of the leptons in kg and the mass-energy x time in $kgm^2/s$ for the photons, but also that all the component factors within these equations have real world meaning. All the component factors within the equations discovered have common simplistic geometric mappings to the physical world as we understand it. Further we find that analysis of the various factors and implicit variables within these equations directly lead to explanations for many of the other observed physical properties of the particles. Thus the equations discovered lead to much more than just their precise target or objective numerical values.

These references to common sense features of the world as experienced by humans added with the cross referencing of other physical property information tend to virtually eliminate the possibility of these equations being happenstance or coincidence.

The common core of structural descriptions found for both particle species is a radial planar energy density pattern. This radial plane is set at right angles to the flight path of the particles, a straight line for the photons and a circular loop for the leptons. This two dimensional feature then immediately answers the mystery of the independence of the photon’s quantum of energy from its wave length. This is in the same sense that the leptons’ rest masses are independent of their activated or excited states. The mathematical nature of this radial planar structure is outlined in the next immediate section A, and discussed in full detail for both particle species in Section IV.A.

The structures detailed in depth in this article describe the gravitational picture of the elementary electromagnetic particles. In the last section of this article we draw an analogy for the photons to the mathematical description of the electromagnetic structure discussed in [1] for the leptons. This mathematical analogy explains why the photons, while they may be “carriers” of charge, display no measurable charge themselves.

A. Mathematical Framework for Structures of Electromagnetic Particles

In this section, we will describe the common mathematical-geometric features found for all the electromagnetic particles, both leptons and photons alike. These are as follows.

1. A radial (2 dimensional, planar) energy density equation, $D_L(r)$ for the leptons and $D_P(r)$ for the photons. Note when used within the article describing the leptons’ mathematics [19], $D_L(r)$ appears as $D_{pk}(r)$, where $p$ designated the electron, muon, or tau and $k$ designated the different energy shells.

2. Within the radial equation there is a Radial Spatial Function or Factor $R_{sf}$ which is identical for the two species leptons and photons.

3. Within the radial equation there is a Radial Temporal Function or Factor $R_{tfL}$ for the leptons and $R_{tfP}$ for the photons. These two have the identical generic appearance or form $R_{sf}$ seen later.

4. Within the Radial Temporal Function there is the embedded or implicit variable $r(t_L)$. This is distinct for the two species and becomes the subject of much discussion. Thus here this variable will be labeled $r(t_{fL})$ and $r(t_{fP})$ when being applied to the specific species.

5. An angular (a single angle) energy density equations, $D_L(\theta)$ for the leptons and $D_P(\theta)$ for the photons. Note when used within the article describing the leptons’ mathematics [19], $D_L(\theta)$ appears as $D_{pk}(\theta)$, where $p$ designated the electron, muon, or tau and $k$ designated the different energy shells.

6. The angular equation has an outer or exterior Spatial Functional appearance $A_{fL}$ for the leptons and $A_{fP}$ for the photons. These two have the identical generic appearance or form $A_{sf}$. This function is based on the Chebyshev $T^T$ orthogonal polynomials.

7. The angular equation has an inner or implicit Temporal Functional appearance $A_{tL}$ and $A_{tP}$ which are again identical in generic appearance or form $A_{tf}$.

8. Within the Angular Temporal Function there is the embedded or implicit variable $\theta(t_L)$. This is distinct for the two species and becomes the subject of much discussion. Thus here this variable will be labeled $\theta(t_{fL})$ and $\theta(t_{fP})$.

9. Initial temporal conditions for both the radial and angular equations, which lead to initial multiplying factors or constants. For the lepton article [19], these are $I(r)$ which leads to the factor $C_{gpk}$ and $I(\theta)$ which leads to the factor $C_{gpk}$. Here we will use the designations $I(r_L)$, $I(r_P)$, leading to $C_{rL}$, $C_{rP}$, respectively, and $I(\theta_L)$, $I(\theta_P)$, leading to $C_{\thetaL}$, $C_{\thetaP}$.

10. A general scale factor or correlation constant for all the particles, composed of basic a-priori measured physical constants. This appears as $C_g$ in the lepton article, and will be designated $C_L$ and $C_P$ here.

11. An overall equation combining the radial equations, angular equations, and the final scale factors as multipliers. This appears as $m_{eL}$ in the lepton article [19], and describes the mass of the particle in kg. Here we will designate this quantity as $m_{eL}$ to describe the “energy” of an electromagnetic particle.

Here we use the word energy, in a very loose sense, to describe the result of these equations. The ultimate result of the lepton equation is mass. Where the ultimate
result of the photon equation is mass-energy x time, i.e. units of the Planck constant.

III. MATHEMATICAL PRELIMINARIES

For this work three mathematical tools or features are of importance. 1. The Fraunhofer Diffraction Function 2. The distance function. 3. The exponential form $e^{-\alpha r^2}$. This last form will be discussed after the findings of this work as applied to the photons have been presented.

First, the less commonly found Fraunhofer Diffraction Function $F_{dfn}[F(r)]$ is first used and discussed in the lepton chapter of [1]. Reviewing, the key features of this function are as follows.

$$F_{dfn}[F(r)] = \left[ \frac{2J_1[F(r)]}{F(r)} \right]^2$$

(1)

Where $J_1$ is the first order Bessel Function of the first Kind. Specifically in this work $F(r)$ is found to be $= k_L r^1$ for the leptons and $= k_P r^{1/2}$ for the photons. Where

$$k_L = 1.697, 525, 53... = \int_0^{\infty} F_{dfn}[1.000,...r^1]dr$$

(2)

Note that $\int_0^{\infty} F_{dfn}[k_L r^1]dr = 1.000, ...$ and thus could be used as a self normalizing initial distribution.

$$k_P = 1.980, 416, 377... = \left( \int_0^{\infty} F_{dfn}[1.000,...r^{1/2}]dr \right)^{1/2}$$

(3)

Note that $\int_0^{\infty} F_{dfn}[k_P r^{1/2}]dr = 1.000, ...$ and thus could be used as a self normalizing initial distribution.

More generalized we can demonstrate that where $F(r)$ is the monomial $= k_n r^n$, if we calculate $k_n$ as,

$$k_n = \left( \int_0^{\infty} F_{dfn}[1.000,...r^n]dr \right)^n$$

(4)

then $\int_0^{\infty} F_{dfn}[k_n r^n]dr = 1.000, ...$. Thus we can always create a candidate for a self normalized initial distribution using the Fraunhofer Diffraction Functions.

Graphical presentations of both of the functions $F_{dfn}[k_L r^1]$ and $F_{dfn}[k_P r^{1/2}]$ can be found in Figures 1 and 2 at the end of the article.

This Fraunhofer Diffraction Function has many interesting properties. For example, for the monomial $= ar^n$ we find a reciprocal scaling property, $\int_0^{\infty} F_{dfn}[ar^n]dr = 1.697, 525.../a$. For calculational purposes, the most important property is the slowing of convergence with decreasing value of n, toward 0, when using the function $F(r) = k_n r^n$.

The second tool, the distance function, is a common heavily used mathematical tool taught in first semester integral calculus. To clarify for the reader the usage of this function here will be in a two dimensional rectilinear setting. Thus:

$$\frac{ds}{dt} = \left[ 1 + \left( \frac{dy}{dt} \right)^2 \right]^{1/2}$$

(5)

Specifically if we have $Y = r(t_r) = (ak_{2n}^{1/2}\pi t_r^2)$, which describes a parabolic curve that happens to be the weighted area of an expanding circle. Then the instantaneous distance along this parabolic curve in rectilinear coordinates is

$$ds(Y) = ds \left( \frac{a\pi t_r^2}{k_{2n}^{1/2}} \right) = \left[ 1 + \left( \frac{2a\pi t_r}{k_{2n}^{1/2}} \right) \right]^{1/2} dt$$

(6)

More generally if $Y = r(t_r) = (ak_n^{-2/2}\pi t_r^{2n})$, then we have the weighted “circular area” producible from an initial curve $(a^{1/2}k_n^{-1/4}t_r^1)$. The instantaneous distance along this “area” figure is

$$ds(Y) = ds \left( \frac{a\pi t_r^{2n}}{k_{2n}^{1/2}} \right) = \left[ 1 + \left( 2naxt_r^{2n-1} \right) \right]^{1/2} dt$$

(7)

IV. APPLICATIONS TO PHYSICAL PROPERTY DETERMINATIONS

A. General Correlation of Mathematical Structural Appearances

The general or generic forms of the mass density equations developed for the leptons is discussed in the lepton chapter of [1]. The specific application of these mathematical forms to the individual leptons to give exact physical property information is also detailed there. Here we wish to compare / contrast structural equation forms for the two electromagnetic species the leptons and the photons. The specific application of these equations to the photons will be given in subsection B.

Beginning with the overall or final equation for calculating the energy of an electromagnetic particle, $e_p$, we have:

$$e_p = C_g C_p D_p$$

(8)

where $C_g$ is a general correlation constant or universal scaling constant for each species. $C_p$ is the specific correlation constant for each lepton member. For the electron this is equal to 1.0. For the photons $C_p$ also is equal 1.0 or may not apply at all. Thus here these two correlation constants will be simplified to a single constant $C_L$ and $C_P$ for the leptons and photons respectively. $D_p$ is immediately composed of a radial equation $D(r)$ and an angular equation $D(\theta)$. Additionally it is summed over
the various applicable shells for the leptons. To maintain clarity of focus here we will simply refer to the mathematical appearances of the electron, which only has one shell. Thus we have specifically for the two species:

\[ e_{\text{Lepton}} = e_L = C_L D_L = C_L D_L(r) D_L(\theta) \quad (9) \]

\[ e_{\text{Photon}} = e_P = C_P D_P = C_P D_P(r) D_P(\theta) \quad (10) \]

where \( e_L \) results in units of mass, \( k_g \), and \( e_P \) results in units of mass-energy \( x \) time, \( \text{kgm}^2/\text{s} \).

Beginning at the beginning;

\[ C_L = e \mu_0 (G\epsilon_0)^{1/2} = 4.893, 752, 96(10^{-36}) \text{m} \quad (11) \]

and

\[ C_P = e^2 (\mu_0 / \epsilon_0)^{1/2} = 9.670, 562, 404(10^{-36}) \text{kgm}^2/\text{s} \quad (12) \]

The units of both these constants need some clarification. Both are actually conversion constants from the common or MKS relative system to units in the Planck absolute system. They come from equations of the form 1.0, relative unit = \( x.xxx(10^{-2}) \), absolute units.

For both species \( D(r) \) is composed of an initial constant, a spatial factor, and an independent temporal factor. This spatial factor, \( R_{sf} \), is a straight function of \( r \) and is identical for the two electromagnetic species. The temporal factor, \( R_{tf} \), contains an embedded or implicit variable of radial time, \( r(t_r) \) and is distinct for the two species. Thus we have:

\[ D_L(r) = C_r L \int_0^{\infty} R_{sf} R_{tfL} dr = C_r L \int_0^{\infty} R_{sf} R(r(t_rL))dr \quad (13) \]

\[ D_P(r) = C_r P \int_0^{\infty} R_{sf} R_{tfP} dr = C_r P \int_0^{\infty} R_{sf} R(r(t_rP))dr \quad (14) \]

The radial initial constants are derived from initial conditions as follows.

\[ I(r) = F_{dfn}[F(r)] = \left[ \frac{2 J_1[F(r)]}{F(r)} \right]^2 \quad (15) \]

Specifically;

\[ I(r_L) = F_{dfn}[k_L r_L] = F_{dfn}[1.697, 525, 53...r_L] \quad (16) \]

Which leads to the initial or normalizing constant for the leptons' radial equation.

\[ C_{rl} = \int_0^{\infty} I(r_L) R_{sf} R(r(t_rL))dr \quad (17) \]

Likewise;

\[ I(r_P) = F_{dfn}[k_P r^{1/2}] = F_{dfn}[1.980, 416, 377...r^{1/2}] \quad (18) \]

Which leads to the initial or normalizing constant for the photons' radial equation.

\[ C_{rP} = \int_0^{\infty} I(r_P) R_{sf} R(r(t_rP))dr \quad (19) \]

Where the derivation of the constants \( k_L \) and \( k_P \) were given in equations 2 and 3 above.

The radial spatial factor, \( R_{sf} \), is the exponential form \( e^{-br^2} \). The generic form of this function, \( e^{-ar^2} \), has many special properties to be discussed later.

The radial temporal factor, \( R_{tf} \) is specific for the two electromagnetic species and derives as follows. For the leptons;

\[ R_{tfL} = e^{r(t_rP) t_m^n (r(t_rL))} \quad (20) \]

Where the \( L_m^n \) is a \( m \)th derivative of the \( n \)th Laguerre orthogonal polynomial. Where both \( n \) and \( m \) are even for the leptons, and specifically \( n = m = 0 \) for the electron. Since \( L_0^0 = 1.0 \) and since the normalization factor for \( L_0^0 \) also is 1.0, then this polynomial factor is implied in the discussions here. This factor is absolutely necessary in correctly calculating and making a distinction between the masses for the higher members of the lepton series. The implicit radial temporal variable for the leptons is

\[ r(t_rL) = \left( \frac{\pi}{2} \right)^{1/2} \int_0^{\infty} \frac{2 \pi t_r^2}{k_L^{3/2}} = \left( \frac{\pi}{2} \right)^{1/2} \left[ 1 + \left( \frac{4 \pi t_r^2}{k_L^{3/2}} \right)^2 \right]^{1/2} \quad (21) \]

Likewise for the photons;

\[ r(t_rP) = \left( \frac{\pi}{2} \right)^{1/2} \int_0^{\infty} \frac{\pi t_r^2}{k_P^{3/2}} = \left( \frac{\pi}{2} \right)^{1/2} \left[ 1 + \left( \frac{\pi t_r^2}{k_P^{3/2}} \right)^2 \right]^{1/2} \quad (22) \]

The Laguerre orthogonal polynomial factor, \( L_m^n \), may not apply at all for the photons. Even if it does apply, just as with the electron this factor is likewise invisible here. The implicit radial temporal variable for the photons is

\[ r(t_rP) = \left( \frac{\pi}{2} \right)^{1/2} \int_0^{\infty} \frac{\pi t_r^2}{k_P^{3/2}} = \left( \frac{\pi}{2} \right)^{1/2} \left[ 1 + \left( \frac{\pi t_r^2}{k_P^{3/2}} \right)^2 \right]^{1/2} \quad (23) \]

Although ultimately in \( r(t_rP) \) the implicit variable \( t_r \) is raised to the 0 power, its appearance is intentionally kept to show a pattern for the two species. The difference between \( t_r \) ultimately raised to the 2nd power for the leptons and \( t_r \) raised to the 0 power for the photons is critical in understanding the physical property differences between the two particles. These differences are discussed in detail later.

For both species \( D(\theta) \) is composed of an initial constant, a symmetry multiplier, and a spatial factor, \( A_{sf} \). This spatial factor is identical for the two electromagnetic species, and in turn contains an embedded temporal factor, \( \theta(t_0) \). The temporal factor contains an embedded or implicit variable of angular time, \( t_0 \). This embedded
temporal function is identical in generic form for the two species.

The outside or primary spatial function within the angular equation \( D(\theta) \) has the form:

\[
A_{sf} = T_0^4(\sin[\pi/2 \, \theta(\theta)])
\] (24)

These are trigonometrically substituted Chebyshev orthogonal polynomials and thus require normalizing factors of \((\pi/2)^{-1/2}\). For both the electron and the photons \( n = 1 \). Thus there is a single \( \sin(\theta) \) term present, which keeps the appearances simple. Thus we have;

\[
D_L(\theta) = C_{\theta L} \ast 4 \left( \frac{\pi}{2} \right)^{-\frac{1}{2}} \int_0^{\pi/2} T_1^4(\sin[\pi/2 \, \theta(\theta)])d\theta
\] (25)

\[
D_P(\theta) = C_{\theta P} \ast 2 \left( \frac{\pi}{2} \right)^{-\frac{1}{2}} \int_0^{\pi/2} T_1^4(\sin[\pi/2 \, \theta(\theta)])d\theta
\] (26)

The angular initial constants are derived from initial conditions as follows.

\[
I(\theta_L) = \cos(\theta)
\] (27)

Which leads to the initial or normalizing constant for the leptons’ angular equation.

\[
C_{\theta L} = \int_0^{\pi/2} I(\theta_L) \ast T_1^4(\sin[\pi/2 \, \theta(\theta)])d\theta
\] (28)

Likewise;

\[
I(\theta_P) = \cos(\pi/8 \ast \theta)
\] (29)

Which leads to the initial or normalizing constant for the photons’ radial equation.

\[
C_{\theta P} = \int_0^{\pi/2} I(\theta_P) \ast T_1^4(\sin[\pi/2 \, \theta(\theta)])d\theta
\] (30)

As seen above in \( D_L(\theta) \) the net symmetric factor for the leptons is 4. This is discussed in the lepton chapter as a result of symmetry of the integral about zero, and as a result of 2 orthogonal forms being applicable. As seen above in \( D_P(\theta) \) the symmetry factor for the photons is 2, which is again the result of symmetry of the integral about zero.

The implicit angular temporal function, \( \theta(t) \) is specific for the two electromagnetic species and is derived as follows. For the leptons;

\[
\theta(\theta_L) = T_0^4(\cos[n^{-1} \, t_0])
\] (31)

where again \( n = 1 \) for the electron.

Likewise for the photons;

\[
\theta(\theta_P) = [1 - (\pi/4 \, t_0)^2]^{1/2}
\] (32)

Since in \( \theta(\theta_L) \) for the leptons the \( \cos() \) function can be restated as

\[
\cos(t_0) = [1 - \sin^2(t_0)]^{1/2}
\] (33)

then this angular temporal variable can be seen to have the same generic or meta-form for both species.

\[
\theta(t_0) = [1 - f^2(t_0)]^{1/2}
\] (34)

Thus we conclude detailing the mathematical forms which describe the bases of the structures of the leptons and the photons. The specific application of these mathematical equations to calculate the masses of the leptons, to the required accuracy, are detailed in the lepton chapter. Here, in the next section, we will walk through the application of these equations to calculate the mass-energy \( x \) time, the Planck constant, for the photon.

Thus we end this section seeing that the mathematics of the structural forms for the two basic or elementary electromagnetic particle species, the leptons and the photons, have analogous and often identical features. The minor differences between their mathematical forms lead to profound differences in their measured and observed physical properties. These differences and their results will be discussed in later sections. The similarities in their mathematics also lead to profound implications not only for these two particle species but for all the other elementary particles and even for cosmology. Discussions of these implications will be left for future articles.

B. Application of Structural Forms to the Photons

We saw that the equations found show the mass-energy \( x \) time, the Planck constant, for the photons to be the mathematically analogous counterpart or resulting structural equivalence to the mass of the leptons. Table I which follows, details the exact use of these structural equations to calculate the Planck constant for the photons. For comparison Table II shows the analogous calculations for the mass of the electron.

We also saw in the last section that the overall radial function for the photons, \( D_P(r) \), was composed of two factors inside the integral; a spatial factor, \( R_{sf} \), and a temporal factor, \( R_{tfP} \) or \( R(r(t_{fP})) \). Further, we saw that although initially \( r(t_{fP}) \) was composed of a monomial of the variable \( t_{fP} \) as follows,

\[
\left( \frac{\pi t_{fP}^2}{k_{P}^{1/2}} \right)
\] (35)

that when the distance function \( ds \) was applied to this monomial, then the variable \( t_{fP} \) effectively vanished. Thus this second factor of time reduces to a constant and the remaining radial spatial factor then can be integrated analytically. This same simplification does not occur with the radial function of the leptons.

Items not detailed in Table I and Table II are the exact forms of the angular implicit temporal variables \( \theta(\theta_L) \) and \( \theta(\theta_P) \) to which are found in equations 31 and 32 above. This is because they are not integrated independently and thus have no free standing numerical values.
TABLE I: Calculation of The Planck Constant

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>SYMBOLS</th>
<th>EXPRESSION</th>
<th>INTEGRATED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Constant</td>
<td>$C_{r,P}$</td>
<td>$\int_0^\infty F_{Q,n}[k_{r,P}^{1/2}]R_{r,F}R_{r,F}d\tau$</td>
<td>6.242, 125, 254</td>
</tr>
<tr>
<td>Spatial Function</td>
<td>$R_{s,f}$</td>
<td>$\int_0^\infty e^{-6r^2} dr = \sqrt{\pi}/(2\sqrt{6})$</td>
<td>0.361, 800, 627</td>
</tr>
<tr>
<td>Temporal Function</td>
<td>$R_{t,f}$</td>
<td>$\int_0^\infty e\left([\frac{\tau}{2}]^{1/2} \left[\frac{\tau+1}{\tau+2}\right]^{2}\right)$</td>
<td>21.451, 225, 729</td>
</tr>
<tr>
<td>Net Integration</td>
<td>none</td>
<td>$\int_0^\infty R_{s,f} R_{t,f} d\tau$</td>
<td>7.761, 066, 925</td>
</tr>
<tr>
<td>Final Value</td>
<td>$D_P(r)$</td>
<td>$C_{r,P} \int_0^\infty R_{s,f} R_{t,f} d\tau$</td>
<td>48.445, 551, 845</td>
</tr>
<tr>
<td>Angular Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial constant</td>
<td>$C_{\theta}$</td>
<td>$\int_0^{4/\pi} \cos(\pi/2 \theta)A_{s,f} d\theta$</td>
<td>0.781, 812, 090</td>
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<tr>
<td>Symmetric Factor</td>
<td>$sym$</td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>Normalizing Factor</td>
<td>$T_1^n$</td>
<td>$\int_0^{4/\pi} T_1^n \left[\sin(\pi/2 \theta \theta \theta \theta)\right] d\theta$</td>
<td>0.797, 884, 561</td>
</tr>
<tr>
<td>Angular Equation</td>
<td>$A_{s,f}$</td>
<td>$\int_0^{4/\pi} T_1^n \left[\sin(\pi/2 \theta \theta \theta \theta)\right] d\theta$</td>
<td>1.133, 648, 187</td>
</tr>
<tr>
<td>Final Value</td>
<td>$D_P(\theta)$</td>
<td>$C_{\theta} * sym * norm \int_0^{4/\pi} A_{s,f} d\theta$</td>
<td>1.414, 329, 947</td>
</tr>
<tr>
<td>Combined Results</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial * Angular Product</td>
<td>$D_P(r)D_P(\theta)$</td>
<td>$\int_0^{4/\pi} \cos(\pi/2 \theta)A_{s,f} d\theta$</td>
<td>68.517, 994, 75kgm²/s, abs</td>
</tr>
<tr>
<td>Scaling Factor</td>
<td>$C_L$</td>
<td>$e^2 (\mu_0 / \epsilon_0)^{1/2}$</td>
<td>9.670, 562, 404(10⁻³⁶), abs/rel</td>
</tr>
<tr>
<td>Final Calculated Value</td>
<td>$e_{Photon}$</td>
<td>$C_L D_P(r) D_P(\theta)$</td>
<td>6.626, 075, 440(10⁻³₄)kgm²/s, rel</td>
</tr>
</tbody>
</table>

TABLE II: Calculation of Electron Mass

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>SYMBOLS</th>
<th>EXPRESSION</th>
<th>INTEGRATED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial Constant</td>
<td>$C_{r,L}$</td>
<td>$\int_0^\infty F_{Q,n}[k_{r,L}^{1/2}]R_{r,F}R_{r,F}d\tau$</td>
<td>1.618, 533, 691(10²)</td>
</tr>
<tr>
<td>Spatial Function</td>
<td>$R_{s,f}$</td>
<td>$\int_0^\infty e^{-6r^2} dr = \sqrt{\pi}/(2\sqrt{6})$</td>
<td>0.361, 800, 627</td>
</tr>
<tr>
<td>Temporal Function</td>
<td>$R_{t,L}$</td>
<td>$\int_0^\infty e\left([\frac{\tau}{2}]^{1/2} \left[\frac{\tau+1}{\tau+2}\right]^{2}\right)$</td>
<td>21.451, 225, 729</td>
</tr>
<tr>
<td>Net Integration</td>
<td>none</td>
<td>$\int_0^\infty R_{s,f} R_{t,L} d\tau$</td>
<td>7.761, 066, 925</td>
</tr>
<tr>
<td>Final Value</td>
<td>$D_L(\tau)$</td>
<td>$C_{r,L} \int_0^\infty R_{s,f} R_{t,L} d\tau$</td>
<td>5.548, 601, 040(10⁴)</td>
</tr>
<tr>
<td>Angular Parameters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial constant</td>
<td>$C_{\theta}$</td>
<td>$\int_0^{\pi/2} \cos(\theta)A_{s,L} d\theta$</td>
<td>0.890, 365, 284</td>
</tr>
<tr>
<td>Symmetric Factor</td>
<td>$sym$</td>
<td></td>
<td>4.0</td>
</tr>
<tr>
<td>Normalizing Factor</td>
<td>$T_1^n$</td>
<td>$\int_0^{\pi/2} T_1^n \left[\sin(\pi/2 \theta \theta \theta \theta)\right] d\theta$</td>
<td>0.797, 884, 561</td>
</tr>
<tr>
<td>Angular Equation</td>
<td>$A_{s,L}$</td>
<td>$\int_0^{\pi/2} T_1^n \left[\sin(\pi/2 \theta \theta \theta \theta)\right] d\theta$</td>
<td>1.180, 580, 070</td>
</tr>
<tr>
<td>Final Value</td>
<td>$D_L(\theta)$</td>
<td>$C_{\theta} * sym * norm \int_0^{\pi/2} A_{s,L} d\theta$</td>
<td>3.354, 777, 477</td>
</tr>
<tr>
<td>Combined Results</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial * Angular Product</td>
<td>$D_L(\tau)D_L(\theta)$</td>
<td>$\int_0^{\pi/2} \cos(\theta)A_{s,L} d\theta$</td>
<td>1.861, 432, 180(10⁵)kg, rel/m, abs</td>
</tr>
<tr>
<td>Scaling Factor</td>
<td>$C_L$</td>
<td>$e\mu_0 (\gamma e\theta)^{1/2}$</td>
<td>4.893, 752, 96(10⁻³⁶)m, abs</td>
</tr>
<tr>
<td>Final Calculated Value</td>
<td>$e_{Lepton}$</td>
<td>$C_L D_L(\tau) D_L(\theta)$</td>
<td>9.109, 389, 239(10⁻³³)kg, rel</td>
</tr>
</tbody>
</table>

V. ANALYSIS AND DISCUSSIONS

A. Initial General Observations

First, we begin at the beginning and ask the question, what do these two tables, Table I and Table II represent in terms of practical physics? The equations in these tables offer straightforward relatively simple mathematical means to calculate measures of the entrapped or stabilized gravitational energy of the two particle species. For the leptons these equations directly result in the units most closely associated with gravity; those of mass, kg. For the photons, for reasons to be discussed more fully later, the equations result in the more indirect units of mass-energy x time, kgm²/s. These equations are in radial planar coordinates, and are represented by regular or scalar mathematics. Both the radial and angular factors
probably represent the solutions to independent second order differential equations. Much valuable information can be learned here, which may give pointers on how to extend the current analysis to other bosons or fermions.

The key operative word here is stabilized or stability. Both the photons and the leptons are stable wave forms. Just because the photons move and the leptons stay put does not diminish the stability of either form. The primary structural feature of both species’ wave forms is a radial plane. Something about the nature of the forces which set up this plane must be self balancing or stabilizing. Stated very generically, or simplistically, the expansive and contractive forces creating these radial structures must be counter-balancing. We know that for these two most elementary electromagnetic particles there are only three forces involved in their structures or to which they respond; gravity, electrical, and magnetic. Thus we can see two of these forces create a 2 dimensional planar structure, while the third propels the wave pattern into space with time. We also saw in the lepton article a different mathematical description, rectilinear vectors, gave the electromagnetic structural picture for the leptons. The probable analogous electromagnetic structural picture of the photons will be discussed later.

In the lepton article [19], we discussed the three factor appearance of the lepton radial equation. There we noted the appearance of a driver, a shaper, and an attenuator. These factors are mathematical representations of what must occur physically. Neglecting the shaping Laguerre polynomial factor, which might not apply to the photons, we found the double exponential appearance of the radial equation for the energy density of the particle to be

\[ D_{\text{particle}}(r) = \frac{1}{C_r} \int_0^\infty R_{sf} R_{sf} dr = \int_0^\infty e^{-ar^2} e^{r(t_\tau)} dr \]  

(36)

The mathematical features of both of these radial factors are quite remarkable.

The equations in Tables I and II are static density equations. Although these equations contain both spatial and temporal components, they are actually static in nature. Looking at the radial equations, \( D_{\text{particle}}(r) \) in particular, we find the spatial, \( R_{sf} \), and temporal, \( R_{tf} \), parameters are multiplied together as independent functions, as seen above.

What we do not find is dynamic, velocity, or first derivative descriptions. If we assume that these descriptive equations are representations of dynamic systems or stabilized wave patterns, then we can ask how does the radial density change with the outward velocity of the wave? Is the outward velocity of the wave constant? We can ask several similar such questions. This information is not addressed by these equations and is not seen directly, if it is available at all. Taking the first derivative of \( D_{\text{particle}}(r) \) simply removes the integral from around \( R_{sf} \) and \( R_{tf} \). Taking yet another derivative, the total second derivative of \( D_{\text{particle}}(r) \) we obtain as follows:

\[ d^2(D_{\text{particle}}(r)) = \frac{d^1(R_{sf})}{dr} R_{tf} + R_{sf} \frac{d^1(R_{tf})}{d(t_\tau)} \]  

(37)

Examining the nature of the terms and factors of this expression for both particle species we can approach some understanding of the interplay of the mass or energy density of these particles with the dynamics of the wave.

Another, yet very important, observation on the nature of these functions concerns their balancing of what could loosely be called "their tendency toward movement". Starting with the radial spatial function, we find that although \( r^2 \) is always positive, the negative sign preceding it in the function \( e^{-6r^2} \) ensures that this radial function or the force it represents is at the top of an energy hump. Thus spatially the internal balance of the particle is such that it would want to "roll" forward or backward. Since the particle is a stable "object", we know that this "tendency toward movement" must be balanced by some other feature of the particle’s energy representation. Here we find that the leptons’ radial temporal function is in a well with very steep walls of \( e^{+6r^2} \). The photon with a flat radial temporal function of \( e^{-6r^2} \) has no such balancing action to prevent spatial movement. Looking at the angular functions, we find that the temporal function is internal or implicit and thus cannot effectively balance either particle species’ external spatial tendency to roll, rotate, or spin about their center.

Finally we need to briefly address the units resulting from these equations. As noted earlier the scaling factors \( C_L \) and \( C_P \) are actually conversion factors relating the Planck system of absolute scales and the MKS system of relative scales. In the unpublished article by Jim Fisher "Systems Analysis - Derivations of essential Constants" the specific operations here have been shown to be derived from numeric quantities with units which are system independent. Thus they are not just mere many decimal accurate coincidences of the MKS relative system of units.

B. Special Properties of Some of The Functions

Several valuable properties of the functions involved in the radial planar structures of the elementary electromagnetic particles are the self-normalization of their free standing initial conditions, and their implicit angular functions, as seen in Table III

Referring back to the mathematical preliminaries, we discussed the application of the distance function, \( ds \), to a general weighted expanding "circular area", \( Y = r(t_\tau) = (ak_{2n}^{-1/2}r^2n^2) \). We saw that this "area" figure was producible from an initial curve \( (a^{1/2}k_{2n}^{-1/4}r_0^2) \). Applying this same concept of a hidden, original, or precursor function to the implicit variables within our radial temporal functions, \( r(t_\tau) \), we find the following. For the leptons, within \( r(t_{sL}) \) the distance function is applied to
2πτ₁²/kr₁¹/² which can be thought of as being the area of a circle produced from √2τ₁/kr₁¹/⁴. For the photons, within r(tₚ,r) the distance function is applied to πτ₂¹/²kr₂¹/² which can be thought of as being the area of a circle produced from τ₂¹/²/kr₂¹/⁴. What we can notice with these “behind the scenes” formulations are that the ultimate variables here have the same power relations as we found for the variables in the radial initial spatial conditions; I(τₗ) with Fₖ[kr₁τ₁] and I(rₚ) with Fₖ[kr₂τ₂¹/²].

Other subtle yet revealing features we find within r(tₗ,τₗ) and r(tₚ,r) are the distinct appearances of kr₁¹/² and kr₂¹/² in the denominators. The appearance 1/(kr₁¹/²) is a classic normalization appearance, where k is the norm.

By norm we mean the integral of some function of concern, squared, over its natural range, when this function is free standing or not embedded in the current application. This is a common meaning, and is the mathematical usage when working with both probability functions and orthogonal polynomials. We can also back the 1/√k out from under the squared term of the distance function, ds, to produce simple the appearance of 1/k. This is also quite a legitimate appearance for a normalization factor, when working with functions that are not squared, such as the Fraunhofer diffraction integrals above.

For the angular functions there is also some subtle interplay between the various functions and their resulting numerical values. In Table II we saw the lepton initial angular constant Cₖₗ evaluated to 0.890, 365, 284... For the photon we can see for a free standing or more general form of the angular function the following properties;

\[ \int_0^1 \sin(\pi/2) \theta (t_{θP}) dθ = \frac{2}{\pi} \]  
\[ \int_0^1 \sin(\pi/2) \left[ 1 - (t_{θ} ∧ u)^2 \right]^{1/2} dθ = 0.890, 365, 284... \]

Further, if we let a = 0.890, 365, 284... b, be any constant, and the upper limit u = b/a, then for all

\[ \int_0^a \sin(\pi/2) \left[ 1 - (t_{θ}/u)^2 \right]^{1/2} dθ = b \]

Thus the integral of the photon angular function self normalizes when we have u = 1/a = 1/0.890, 365, 284...

### Table III: Integrals of Free Standing Functions Used in cₚ

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>SYMBOLS</th>
<th>INTEGRATED EXPRESSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton radial initial condition</td>
<td>I(tₗ)</td>
<td>[ \int_0^{τₖ} F_{df}[1.697, 525, 535τ] dτ = 1.0 ]</td>
</tr>
<tr>
<td>Photon radial initial condition</td>
<td>I(rₚ)</td>
<td>[ \int_0^{τₖ} F_{df}[1.980, 416, 377τ]^{1/2} dτ = 1.0 ]</td>
</tr>
<tr>
<td>Lepton angular initial condition</td>
<td>I(θₗ)</td>
<td>[ f_0^{π/2} Cos(θ) dθ = 1.0 ]</td>
</tr>
<tr>
<td>Photon angular initial condition</td>
<td>I(θₚ)</td>
<td>[ f_0^{π/2} Cos(\pi/8 \ θ) dθ = 1.0 ]</td>
</tr>
<tr>
<td>Lepton implicit angular function</td>
<td>θ(tₗ)</td>
<td>[ \int_0^{T_{θₗ}} Cos[n^2 t_{θₗ}²] dt = 0 ]</td>
</tr>
<tr>
<td>Photon implicit angular function</td>
<td>θ(tₚ)</td>
<td>[ \int_0^{(π/4 \ t₀)^2}^{1/2} dτ = 1.0 ]</td>
</tr>
</tbody>
</table>

### Table IV: Result of Sign Variations of Equation Parameters for Leptons

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>EXPRESSION</th>
<th>Rₛᶠ</th>
<th>Rₛˡ</th>
<th>Aₛᶠ</th>
<th>θₛˡ</th>
</tr>
</thead>
<tbody>
<tr>
<td>ds in r(tₗ,τₗ)</td>
<td>[ 1 + \left( \frac{2πτ₁}{kr₁} \right)^2 ]^{1/2} dt</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Rₛˡ</td>
<td>e^{-k'r} \left( \frac{π/2}{kr₂} \right)^2</td>
<td>+</td>
<td>NA</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Aₛᶠ</td>
<td>Tₗ[π/2 \ (θ(tₗ,τₗ))]</td>
<td>+</td>
<td>NA</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>θₛˡ</td>
<td>Tₗ[Cos[n^2 t_{θₗ}²]]</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

1.123, 134, 536... or in effect is linearly scaling. This property can be very useful.

### C. Parity

Before examining the key differences between the equations which give rise to the observed physical properties of the photons and the leptons, we should first look at one other important topic, parity. Without structural equations describing the various particles, physics has proceeded by observing what occurs to the various particles when time is reversed, when space is reflected, et cetera. Physics then makes a tabulation of these observations and notes where expected reflection outcomes are violated. Without a knowledge of structure though, there can be no general mathematically based explanation for the various outcomes. Here we can directly examine what to expect when the basic human measuring devices of time and space are negated or reflected. Tables IV and V summarizes these results.

When looking at the effects of parameter sign variation, we find with only one exception that it is the implicit temporal variable and not the external function that drives the nature of these responses. For the radial spatial functions which are identical as Rₛᶠ, of course there aren’t any temporal variables. This concept of the temporal being a-priori when viewing the nature of particle parity is in agreement with the findings of [20].

Here we find a conceptual difference between the mathematical and the physical. When thinking of trigonomet-
Table V: Result of Sign Variations of Equation Parameters for Photons

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>EXPRESSION</th>
<th>$r,t,\theta,t_0$</th>
<th>$r,t,\theta,t_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{sf}$</td>
<td>$e^{-\alpha r^2}$</td>
<td>$\pm$</td>
<td>$\pm$</td>
</tr>
<tr>
<td>$\theta(t_0)$</td>
<td>$T_n\sin(\pi/2 \theta(t_0))$</td>
<td>$\pm$</td>
<td>$\pm$</td>
</tr>
<tr>
<td>$A_{sf}$</td>
<td>$T_n^2/2$</td>
<td>constant</td>
<td>constant</td>
</tr>
<tr>
<td>$R_{tP}$</td>
<td>$e^{(\frac{\pi}{2})/2}$</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

The electromagnetic structures of these particles to make sense of the dynamics of this particular parameter.

VI. STRUCTURAL CONTRASTS AND CONCLUSIONS

A. Mass - Masslessness

Examining the radial function for the photon and leptons in Tables I and II we find the key difference between the two species, their gravitational relationship to time. From the discussion in Section V.B. above we saw that the outer most level of time, or consensus time on which the distance function $ds$ is operating could be conceived of as having some inner, hidden, or preliminary function of time. The interior relations with time of the two species had the same power relationships as those of their initial spatial Fraunhofer Diffraction conditions. The exterior spatial temporal function of these energy density equations supply one of the factors for the ultimate mathematical values for these expressions. In this temporal function the leptons have a pseudo first-order relation to time, $r(t_P) \propto t^1$ or $\left[1 + \left(\frac{\pi t_0}{k^2} \right)^2 \right]^{1/2}$. Whereas from the external viewpoint the photon is related to time as $r(t_L) \propto t^0$, or $\left[1 + \left(\frac{\pi t_0}{k^2} \right)^2 \right]^{1/2}$, or is mathematically independent of time. This does not mean that the photon has no relationship to consensus time, or a null or zero relationship. Rather the photon has a very special relationship, that of a constant value of 1.0 or more correctly a constant value of 2.446, 139, 955. This is probably the mathematical reason why the photon has no mass and why it travels at the speed of light, $c$; as the speed of electromagnetic energy transfer is called. This is in keeping with relativity which indicates any "object" moving at the speed of light effectively will not age or experiences an unchanging relationship with time, at least from the viewpoint of an external or stationary observer.

From the inner most or precursor view, we saw the photon is still related to time as $t_L^1$. We can assume that this is necessary for the photon to exist. Generalized, all "objects" that exist must have a form or structure. That structure must have internal references to time just as they have external references to space.

B. Charge - Chargelessness

As to why the photon has no charge, or at least why it displays no measurable charge. It is hard to prove a negative, the absence of something, or why something does not, or appears to not, exist. Never-the-less, the following explanation is offered for discussion. The one useful item of information that we have on this topic...
is the mathematical nature of the charge of the leptons discussed in [1]. There it was shown that the charge of the leptons probably relates to the square of the curvature of a vector formulation for the electromagnetic structure of the leptons.

The vector formulation of a broadly generalized "cylindrical spiral" is

\[ \mathbf{R}(t) = a\cos[F(t)]\mathbf{i} + a\sin[F(t)]\mathbf{j} + bG(t)\mathbf{k} \]  \hspace{1cm} (41)

With the one constraint that \( F(t) = G(t) \) then the curvature and torsion of this figure are

\[ \kappa = \frac{[\mathbf{R}'(t) \times \mathbf{R}''(t)]}{|\mathbf{R}'(t)|^3} = \frac{a}{a^2 + b^2} \]  \hspace{1cm} (42)

\[ \tau = \frac{[\mathbf{R}'(t) \times \mathbf{R}''(t) \cdot \mathbf{R}'''(t)]}{|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2} = \frac{b}{a^2 + b^2} \]  \hspace{1cm} (43)

These are invariant numerical constants, independent of the variable \( t \) and the function \( F(t) \). This invariance was shown to be key in producing the invariant vector based charge of the leptons. The equation which calculated to produce the charge of the leptons as, \( e_{\text{calc}} = 1.602,177,29(10^{-19}) \) C, based off the cylindrical helix form

\[ \mathbf{R}(t)_{e/mL} = aT_n^1[\cos(F(t))]\mathbf{i} + aT_n^1[\sin(F(t))]\mathbf{j} + bG(t)\mathbf{k} \]  \hspace{1cm} (44)

for \( n \) odd. Where \( F(t) = G(t) = \pi/2 \: \theta(t_{\phi L}) \). Where \( \theta(t_{\phi L}) = T_n^1[\cos(n^{-1}t_{\phi})] \) with \( a = 6 \) and \( b/n = 1, n = 1, 3, 5 \) for electron, muon, and tau respectively.

Here the function \( F(t) \) did not need to be explicitly stated or even known at the time. Later in the radial angular mass density equations, gravitational structure, we found this angular function to be as shown. If we relate the curvature \( \kappa \) to charge and the torsion \( \tau \) to hand, as was done in [1], then by varying the signs of \( a \) and \( b \) in \( \mathbf{R}(t) \) we can produce four combinations of charge and hand for the lepton and their anti-particles. Allowing space to freely rotate about the particle reduces this to two combinations of charge and hand in a free floating environment.

Thus, although not directly provable by the mathematics of this report, it is felt that the photon violates the one constraint that \( F(t) = G(t) \). By analogy to the leptons, it is likely that the electromagnetic vector formulation for the photon is something similar to the following.

\[ \mathbf{R}(t)_{e/mP} = a\cos[F(t)]\mathbf{i} + a\sin[F(t)]\mathbf{j} + bG(t)\mathbf{k} \]  \hspace{1cm} (45)

Where \( F(t) = \pi/2 \: \theta(t_{\phi P}) \) and \( G(t) = t \); ie. \( F(t) \neq G(t) \). Where \( \theta(t_{\phi P}) = [1 - (n^{-1}t_{\phi})^2]^{1/2} \), and probably with \( a = 6 \) and \( b = 1, n = 4/\pi \).

The photon may spin or rotate clockwise or counterclockwise about the centerline of its axis of progression, related to the sign of \( a \), and it may progress in a forward or backward sense, related to the sign of \( b \). Even so, we find the photon will not have a charge nor a sense of hand because it does not have a fixed invariant curvature nor likewise a fixed invariant torsion. What we really see is that the photon does in fact have a charge, but it is constantly changing. Thus it does not display a charge, a fixed constant physical property which is measurable. To humans and their instruments the photons appear chargeless, at the gross time scale upon which humans operate.

We know that this variable \( t \) in the vector helix expression of the photon relates to position along the wavelength or flight path of the photon. By analogy we can conclude, this variable \( t \) for the lepton relates to position along the donut coil flight path of the lepton. In [1] we concluded that this coil length \( t \) was independent from both the radial and angular expressions of time, \( t_r \) and \( t_{\phi} \), for the rotating or spinning mass density planar pattern. We can assume that the final expressions in Tables I and II, of mass-energy \( x \) time for the photon, and mass for the leptons, relate to time at the outer most or exterior level. Then another key to unscrambling this mystery of charge, or lack of it, will be in determining to what level of the variable \( t \) in their respective electromagnetic structures, vector cylindrical expressions, is related.

Examining these vector expressions for the electromagnetic structures of these particles we can address the parameter of propagation, which was left open by their radial planar gravitational structures. Imagine what we would see if humans had both the conceptual and physical apparatus to see into time. Further imagine that an observer had shrunk down to the size of a photon and watched one move across his front. Assume that as he is standing focusing forward, that he sees the present in time. With the specified propagation vector of \( bG(t)\mathbf{k} \) and \( G(t) = t \) for the photon what would he see as time progressed or regressed? If he looks to the left in time, call this the past, he sees the photon moving in one direction. If he looks to the right in time, call this the future, he sees the photon moving in the other direction. Since this variable in time is so simple and immediately translates into the spatial parameter, our observer would find the same situation in space. If he looks to the left, negative values in space, he finds the photon progressing toward him. If he looks to the right in space, positive values, he finds the photon receding away from him. This is all very simple and as expected.

What is not simple is this same analysis for the lepton circulation around the donut. With the lepton parameter of \( G(t) = \pi/2 \: T_n^1[\cos(n^{-1}t_{\phi})] \) we find very different progression dynamics. Here we need to remember that the variable \( t_{\phi} \) and the parameter \( \cos(n^{-1}t_{\phi}) \) are limited by the valid range of the \( T_n^1 \) function. Here we are assuming that the electromagnetic \( T_n^1 \) function of the leptons is a \( T_n^1 \) function the same as the gravitational one, and that it is not a more general full range \( T_n \) function. As discussed in Subsection V.C. concerning parity for the gravitational structure variables, under this restriction the \( \cos() \) func-
ion always produces a positive value. Thus as our observer views the curved rim of the lepton donut, to his left or to his right, he finds the energy pattern progressing in the same direction. Whether this is negative toward him or positive away from him doesn’t matter. It is doing the same on both sides of him in time. Again since this temporal variable is so simple, movements here in either the past or future immediately translate to their spatial analogues. How can the radial planar pattern progress either away from our observer in both directions or toward him from both directions? This is possible under one set of conditions, that we have two waves, one forward and one reverse, and that they pass thru each other at the point of our observer. This is a plausible possibility since we have specified that our observer is at the present, the point of our observer. This is a plausible possibility since at any other point in time our observer cannot see equally well into the past and into the future. Also his view to the right and left in space will not be identical, but probably will be two mirror images. This explanation may help alleviate some of the apparent anomaly that was raised in Subsection V.C.

<table>
<thead>
<tr>
<th>TABLE VI: Spin of Leptons and Photons</th>
</tr>
</thead>
<tbody>
<tr>
<td>FUNCTION</td>
</tr>
<tr>
<td>$\theta(t_0)$</td>
</tr>
<tr>
<td>$\theta(t_0)^{\prime}$</td>
</tr>
<tr>
<td>$\theta(t_0)\theta(t_0)^{\prime}$</td>
</tr>
<tr>
<td>Equivalent</td>
</tr>
<tr>
<td>Integrated</td>
</tr>
<tr>
<td>Eval @ upr</td>
</tr>
<tr>
<td>Eval @ 0</td>
</tr>
<tr>
<td>Net</td>
</tr>
</tbody>
</table>

C. Spin

As to spin or intrinsic angular momentum, again there is not much information with which to work. Never-the-less we can proceed by making a few simple assumptions. First we assume that angular momentum has to do with the radial planar form of the gravitational structures presented in Tables I and II. We assume that these radial planar forms are spinning in a sense similar to the turning of the blades on a windmill. We assume that this angular momentum has nothing to do with or is independent of the forward propagation of these figures, a straight line for the photon and a circular donut for the lepton. In terms of the electromagnetic vector expressions, we would say that the angular momentum is related to the i and j vectors and is independent of the k vector. Next we assume that angular momentum has only to do with the angular equations of these structures and is not related to the radial equations. Finally we choose a very simplistic or basic concept for the mathematical meaning of momentum. We assume that momentum is related to velocity or the first derivative of a function of time. Specifically we choose the following generic expression for momentum,

$$Momentum, MO[F(r,t)] = F(r,t) \ast F'(r,t) \ (46)$$

Looking at the angular mass density equations, $A_{nf}$, in 24 and Tables I and II we see that both particles have the same form, as follows; $A_{sf} = T_n^0 (\sin[\pi/2 \theta(t_0)])$. Further for $n = 1$, $A_{sf} = \pi/2 \cos[\pi/2 \theta(t_0)]\theta'(t_0)$ Finally, we find:

$$A_{sf}A_{sf}' = \pi/2 \sin[\pi/2 \theta(t_0)]\cos[\pi/2 \theta(t_0)]\theta'(t_0) \ (47)$$

This information is of course identical with what would be obtained if we had started with the j vector of the electromagnetic formulation. We know that the particles, photons being bosons and leptons being fermions, must have a spin angular momentum ratio of 2 : 1. We can rapidly see that these expressions with their embedded implicit functions and derivatives, $\theta(t_0)$ and $\theta'(t_0)$, are sufficiently complicated and substantially different enough that we will not be able to produce this simple 2:1 ratio, no matter how we try to manipulate them.

Thus we are forced to look deeper. While this next step initially appears difficult to rationalize, upon examination we find that it is the only proper logical choice. We must assume that the angular momentum of these particles relates only to the inner, implicit, or embedded functions of time. This last step or assumption produces the desired simplification and the ultimate desired result. Why can we make this last choice excluding the greater or exterior angular equations as the starting point for our computations? Upon examination the answer is obvious. The external angular equations are functions of $\theta$ and space and are not functions of time. We already decided as part of our definition of momentum that an expression representing momentum must be a function of time. Examining the inner, implicit, or temporal angular expressions for our particles we find the information in Table VI. For both species $k = \pi/2$.

Here we need to remember that the lepton angular equations are most probably the solutions to second order differential equations and thus are the trigonometrically substituted $T_n^0$ orthogonal polynomials. Thus the upper limit on the lepton integral of $\theta(t_0)\theta(t_0)'$ is such that it gives the full range of the $T_n^0$ polynomials, 0 to 1, and here is specifically set at $\pi/4$. Here the value of u in the photon equations does not need to be specified, but only needs to be set equal to the upper limit of integration. From the angular equation we know that this upper limit is probably equal $4/\pi$. The simplification of the lepton presentation, which shows only the electron information, still needs to be addressed. The use of higher member odd $T_n^0$ polynomials was neglected here because of possible complications which could be added by the auxiliary shells of the higher members of the lepton series.
Finally, the above presentation of course only shows how to obtain values related to angular momentum which have the correct numerical ratio. There is no scaling factor or correlation constant which converts these values to real world measured quantities. Thus these manipulations can be thought of as only a relative correlation, and are not truly equations from which spin angular momentum can be calculated. The importance of this proposed explanation, though, again lies in verifying that the equations developed for both particles, not only meet their respective numerical requirements, they also meet key conceptual requirements. That is, they can potentially explain obvious physical property differences between the two species.

D. Conclusions

Summarizing, we have seen a mathematical framework for describing geometric structures for the elementary electromagnetic particles, leptons and photons. These wave structures are simple and easily understood by anyone familiar with second semester calculus. These structures base on two dimensional radial-angular energy density patterns, in scalar mathematics, to describe the gravitational picture of these particles. Additionally vector mathematical descriptions of the same structures give the electromagnetic picture of the particles.

We have found that an amazing amount of information has been obtained. Once equations have been developed, discovered for the gravitational structure of these two particle species, leptons and photons, then indeed these equations explain much more than just the observed mass or mass-energy x time for the particles. Each factor in these equations offers interesting insights into other observed physical properties of the particles.

1. The implicit temporal function of the radial equation, $R_{tf}$, offers a plausible explanation for a particle’s masslessness.

2. The implicit temporal function of the angular equation, $A_{tf}$, offers a clear explanation for a particle’s spin angular momentum.

3. The relation between the implicit temporal functions, $F(t)$ and $G(t)$, of the vector description of the particle’s electromagnetic structure offers an explanation for the particle’s charge - chargelessness.

4. Varying the sign of the several implicit and temporal functions gives results for the external radial and angular functions in agreement with the observed parity requirements for these particles.

In short it is the temporal functions or factors in the particle’s structural equations which provide insight into many of the major observed physical properties of the particles.

Thus it appears that through continued analysis of mathematical statements for the structures of both the photon and the leptons that the definitive answers to other physical property questions will be found. Questions such as what gives rise to or is responsible for the magnetic moment of the leptons? Likewise, for the neutrinos, gravitons, quarks, gluons, et cetera, structural equations (gravitational, electromagnetic, and color where applicable) must be developed before much further progress can be made in particle physics. Assuming structureless particles such as mathematical points or mathematical lines with no cross section, and possibly even mathematical sheets with no thickness, will not yield physical property information.

VII. APPENDIX

In Section III the mathematical preliminaries we introduced the exponential form $e^{-ar^2}$. In Section IV.A. we saw the use of this form as the identical primary spatial factors, $R_{sf}$, within the radial functions $D_t$ for both the leptons and the photons. In Section V.A and Tables I and II we saw the importance of this spatial factor in balancing the expansive functions $R_{sf \times} r(t_r) \propto t^0$ and $R_{sf \times} r(t_r) \propto t^1$. Since these temporal functions are embedded within a positive exponential then there needs to be some form of counter balancing mathematical factor to bring the overall radial function to converge, or else these mathematics could not result in a stable wave pattern or physics particle. Aside from terminating the radial temporal tendency of the particle we find some very interesting properties of the form of this spatial factor $R_{sf} = e^{-ar^2}$ by itself. Probably the most important property for our applications to physics is outlined following.

This property concerns the balancing of the traditional forces, potential and kinetic, or their mathematical energy expressions. We need to start with a generic binary expression for the force or energy expression of a system, such as $F(r,t) = G(r)H(t)$. For simplicity of discussion and to illustrate the special property of this primary radial factor, a very specific form has been chosen. $F(r) = 1/2 mR^2 = 1/2 mR(t)^2$, where $t$ is implicit within $r = R(t)$. Here the binary feature is hidden as the second power of a single variable, rather than being shown as the product of two first order variables. Taking the derivatives of this expression, using the chain rule, we find the following:

$$ F(r) = 1/2 \ mR(t)^2 \quad (48) $$

$$ F'(r) = mR(t) \cdot d^1 R(t) \quad (49) $$

$$ F''(r) = mR(t) \cdot d^2 R(t) + m(dR(t))^2 \quad (50) $$

$$ F'''(r) = mR(t) \cdot d^3 R(t) + 3md^1 R(t) \cdot d^2 R(t) \quad (51) $$

If we let $m$ stand for mass and $t$ for time, then here $F'$, the first derivative expression, is traditionally thought of
as an expression involving velocity and as representing momentum. The second derivative expression is what equates to energy in this consensus world and is the subject of almost all engineering and physics calculations. The $d^2$ expression is thought of as acceleration and this term is associated with potential energy. The $(d^4)^2$ expression is thought of as velocity squared and this term is related to kinetic energy. For any stable system with no outside energy input and with no output of energy to the exterior environment, or with no accumulation or depletion of internal energy, these two terms of the $F''$ expression must balance to zero. The third derivative expression is basically ignored as not being relevant to most physical applications.

Taking the indicated derivatives of our radial spatial function $R_{sf} = e^{-ar^2}$, we find

$$F''(r) \text{ or } D^2 = e^{-ar^2}[-2a + 4a^2r^2]$$

(52)

On integrating we find

$$\int_0^\infty F''(r) dr = 0$$

(53)

regardless of the value of the constant a. This is as required for any stable system, or particle in this case. In this illustration of course the expression $F(r)$ is free standing or independent of any additional radial temporal factors. Of even further interest, the $\int_0^\infty$ of all even derivatives of this particular $F(r)$ balance to zero. This means that if we chose any even derivative expression of this $F(r)$ to be the original function, then the integral of this chosen function’s second derivative would also produce a stable balance of forces or energies.

VIII. REFERENCES


See Section III. in the text for detailed discussion.
See Section III. in the text for detailed discussion.